with

$$\alpha r_n = \begin{cases} 0.088\,969, & \text{if n=1} \\ 0.011\,57, & \text{if n=2} \\ 0.019\,70, & \text{if n=3} \end{cases}$$

$$\beta r_n = \begin{cases} 0.4450, & \text{if n=1} \\ 0.3985, & \text{if n=2} \\ 0.4277, & \text{if n=3} \end{cases}$$

$$\gamma r_n = \begin{cases} 0.6189, & \text{if n=1} \\ 0.6039, & \text{if n=2} \\ 0.5178, & \text{if n=3.} \end{cases}$$

The values of the parameters αr_n, βr_n, and γr_n were determined numerically, in order to minimize the average error of the approximate roots.

The exact values of the roots can now be calculated by the method of [4] using as interval for the search of the roots xr_{nm}: $[xr_{1m}^{ap} - .4(xr_{2m}^{ap} - xr_{1m}^{ap}),\, xr_{1m}^{ap} + .4(xr_{2m}^{ap} - xr_{1m}^{ap})]$, for $n = 1$, $[xr_{nm}^{ap} - .4(xr_{nm}^{ap} - xr_{n-1,m}),\, xr_{nm}^{ap} + .4(xr_{nm}^{ap} - xr_{n-1,m})]$, for $n = 2$ and $n = 3$, and $[xr_{n-1,m} + .4(xr_{n-1,m} - xr_{n-2,m}),\, xr_{n-1,m} + 1.4(xr_{n-1,m} - xr_{n-2,m})]$ for $n \geq 4$.

The roots of the denominator of the function S are the solution of:

$$J_m(\delta x)Y_m(x) - J_m(x)Y_m(\delta x) = 0. \tag{A3}$$

It should be observed that this equation is the same as the characteristic equation for TM modes in a coaxial circular waveguide.

The procedure to determine the roots xs_{nm} is the same as applied to the function R. The method of [4] is again used, with the same intervals defined above for R, but replacing xr_{nm}^{ap} and xr_{nm} by xs_{nm}^{ap} and xs_{nm}, respectively. The values of xs_{nm}^{ap} are given by:

$$xs_{nm}^{ap} = \sqrt{(c_3 xs_{nm}^{<})^2 + (c_4 xs_{nm}^{>})^2}, \quad n = 1, 2, 3, \quad 0 \leq m \leq 50 \tag{A4}$$

with

$$c_3 = (1. - \delta)^{\alpha s_n} \quad c_4 = \left(\frac{2\delta}{1+\delta}\right)^{\beta s_n (m+1)^{\gamma s_n}}$$
$$xs_{nm}^{<} = p_{nm} \qquad xs_{nm}^{>} = \frac{n\pi}{1+\delta}$$

$$\alpha s_n = \begin{cases} -0.002\,591, & \text{if n=1} \\ 0.015\,33, & \text{if n=2} \\ 0.024\,62, & \text{if n=3} \end{cases}$$

$$\beta s_n = \begin{cases} 0.2853, & \text{if n=1} \\ 0.4413, & \text{if n=2} \\ 0.4068, & \text{if n=3} \end{cases}$$

$$\gamma s_n = \begin{cases} 0.8402, & \text{if n=1} \\ 0.5396, & \text{if n=2} \\ 0.5014, & \text{if n=3.} \end{cases}$$

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Analytical Behavior of the Noise Resistance and the Noise Conductance for a Network with Parallel and Series Feedback

Luciano Boglione, Roger D. Pollard, and Vasil Postoyalko

An analysis is presented of the changes of the noise parameters of a two-port network when noisy series and parallel feedback immittances are applied. Exact formulas for the noise parameters R_n, g_n, and ρ_n are given as functions of the feedback for a given network. It is proved that R_n always reaches a minimum when a reactive series feedback is considered. The same results are demonstrated for g_n since a duality principle is pointed out. The results are valid for a wide range of linear microwave two-port networks, either passive or active, and they are used to confirm the data from previously published work.

***Index Terms*—Amplifier noise, feedback amplifiers, feedback circuits, microwave amplifiers, noise.**

I. INTRODUCTION

In [1], some guidelines are outlined for feedback amplifier design. The resistive parallel feedback has been investigated by [2] and [3]. The change of the noise figure in the case of either parallel or series feedback was worked out by [4]. In [5], series and parallel feedback are analyzed in order to get simultaneously optimum noise and good input/output standing-wave ratio (SWR). In [6], monolithic technology to fabricate a series feedback amplifier in order to get good repeatability during fabrication and the simultaneous noise match and optimum input SWR is applied. Both simulation and experimental validation of an *X*-band monolithic four-stage low-noise amplifier with series feedback is carried out in [7]; however, the paper does not detail how the simulation has been carried out.

This paper generalizes the results of [6] and [7] using a procedure similar to [1], provides a mathematical tool to investigate the signal

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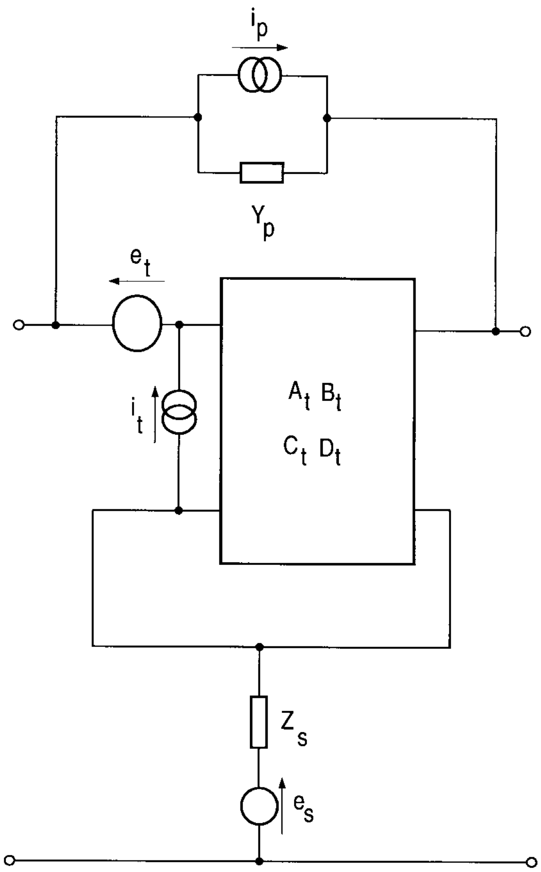

Fig. 1. Schematic of a noisy two-port with series and parallel feedback.

and noise behavior of a feedback network, and presents exact and explicit formulas of the noise parameters for a network with both parallel and series noisy feedback at a given frequency. The approach is not related to any particular technology; the only requirement is a knowledge of the signal matrix and the noise parameters at a given frequency.

II. The Signal and Noise Linear Analysis

Consider the linear circuit in Fig. 1 at a constant frequency f_o whose elements are as follows:

1) black box (typically an active device) characterized by its noise parameters $R_t,\ g_t, \rho_t$ [8], and a signal matrix (as the scattering or the transmission $A_t B_t C_t D_t$ matrix);
2) parallel admittance $Y_p = G_p + jB_p$;
3) series impedance $Z_s = R_s + jX_s$.

Z_s and Y_p are uncorrelated noise sources, modeled by e_s and i_p, respectively [9].

The subscript *t* refers to the active network, *s* to the series feedback, *p* to the parallel feedback, and *n* to the overall network. Assume the active network is represented by its impedance matrix $\mathbf{Z_t}$. The series feedback element can be added directly: $\mathbf{Z_t} + \mathbf{Z_s}$ where $\mathbf{Z_s} = Z_s \mathbf{U_s},\ \mathbf{U_s} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Then, the sum matrix is inverted and the parallel feedback matrix is added:

$$\mathbf{Y_n} = \mathbf{Y_p} + (\mathbf{Z_t} + \mathbf{Z_s})^{-1} \tag{1}$$

where $\mathbf{Y_p} = Y_p \mathbf{U_p}$ and $\mathbf{U_p} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. $\mathbf{Y_n}$ is the admittance matrix of the overall circuit.

A similar procedure can be followed in order to obtain the noise parameters [10] and [11]. Let the following matrices be defined:

$$\mathbf{C_t} = \begin{bmatrix} \overline{e_t\, e_t{}^*} & \overline{e_t\, i_t{}^*} \\ \overline{i_t\, e_t{}^*} & \overline{i_t\, i_t{}^*} \end{bmatrix} \tag{2}$$

$$\mathbf{C_s} = \begin{bmatrix} \overline{e_s\, e_s{}^*} & \overline{e_s\, e_s{}^*} \\ \overline{e_s\, e_s{}^*} & \overline{e_s\, e_s{}^*} \end{bmatrix} = \Re e[\mathbf{Z_s}] \tag{3}$$

$$\mathbf{C_p} = \begin{bmatrix} \overline{i_p\, i_p{}^*} & -\overline{i_p\, i_p{}^*} \\ -\overline{i_p\, i_p{}^*} & \overline{i_p\, i_p{}^*} \end{bmatrix} = \Re e[\mathbf{Y_p}] \tag{4}$$

where * denotes the complex conjugate and the overbar the statistical average. It is tacitly assumed that all noise powers, hence the matrices, are normalized to $4kT_o\Delta f$. The impedance form [10] of the noise matrix of the active circuit is obtained:

$$\mathbf{C_t^{(z)}} = \mathbf{T}_{(t\to Z)} \mathbf{C_t} \mathbf{T}_{(t\to Z)}{}^+ \quad \text{where: } \mathbf{T}_{(t\to Z)} = \begin{bmatrix} 1 & -\frac{A_t}{C_t} \\ 0 & -\frac{1}{C_t} \end{bmatrix}.$$

A_t and C_t are elements of the transmission matrix of the active circuit and $^+$ indicates the Hermitian conjugation. $\mathbf{T}_{(P\to Q)}$ is the transformation matrix from the *P* to the *Q* network representation [10].

The noise matrix (3) of the series feedback impedance is added:

$$\mathbf{C_z} = \mathbf{C_t^{(z)}} + \mathbf{C_s}.$$

Converting this to admittance form and adding to it the noise matrix of the parallel feedback (4) we obtain the admittance form of the noise matrix for the complete circuit. Thus:

$$\mathbf{C_y} = \mathbf{T}_{(Z\to Y)} \mathbf{C_z} \mathbf{T}_{(Z\to Y)}{}^+ + \mathbf{C_p} \quad \text{where: } \mathbf{T}_{(Z\to Y)} = (\mathbf{Z_t} + \mathbf{Z_s})^{-1}.$$

Converting the admittance form to the *ABCD* matrix form:

$$\begin{aligned} \mathbf{C_n} =& \mathbf{T}_{(Y\to T)} \mathbf{C_y} \mathbf{T}_{(Y\to T)}{}^+ \\ =& \mathbf{T}_{(Y\to T)} \mathbf{C_p} \mathbf{T}_{(Y\to T)}{}^+ + \mathbf{T}_{(Z\to T)} \mathbf{C_s} \mathbf{T}_{(Z\to T)}{}^+ \\ &+ \mathbf{T}_{(t\to T)} \mathbf{C_t} \mathbf{T}_{(t\to T)}{}^+. \end{aligned}$$

$\mathbf{C_n}$ is formed by the sum of the contributions from the parallel feedback (first term), from the series feedback (second term), and from the active network (third term). Here,

$$\begin{aligned} \mathbf{T}_{(Y\to T)} &= \begin{bmatrix} 0 & -\frac{1}{Y_{n21}} \\ 1 & -\frac{Y_{n11}}{Y_{n21}} \end{bmatrix} \\ \mathbf{T}_{(Z\to T)} &= \mathbf{T}_{(Y\to T)} \mathbf{T}_{(Z\to Y)} \\ \mathbf{T}_{(t\to T)} &= \mathbf{T}_{(Z\to T)} \mathbf{T}_{(t\to Z)} \end{aligned}$$

and $Y_{n_{ij}}$ are terms of (1).

The four noise parameters can be expressed in terms of the matrix elements of $\mathbf{C_n}$ [10]:

$$\mathbf{C_n} = \begin{bmatrix} \overline{e_n\, e_n{}^*} & \overline{e_n\, i_n{}^*} \\ \overline{i_n\, e_n{}^*} & \overline{i_n\, i_n{}^*} \end{bmatrix} = \begin{bmatrix} R_n & \rho_n{}^\star \sqrt{R_n g_n} \\ \rho_n \sqrt{R_n g_n} & g_n \end{bmatrix}. \tag{5}$$

The expansion of (5) gives (see (6)–(8) at the bottom of the next page), where

$$\begin{aligned} r_1 =&\ g_t \mid a \mid^2 + R_t \mid C_t \mid^2 + 2\,\Re e[a\rho_o C_t{}^\star] + \mid \Delta_o \mid^2 G_p \\ r_2 =&\ \mid a \mid^2 + 2\,\Re e[a\rho_o] + 2R_t\, \Re e[C_t] + 2\,\Re e[\Delta_o B_t{}^\star]\, G_p \\ r_3 =&\ -2\,(\Im m[C_t R_t + a\rho_o] + \Im m[\Delta_o B_t{}^\star]\, G_p) \\ r_4 =&\ \mid B_t \mid^2 \\ g_1 =&\ R_t \mid d \mid^2 + g_t \mid B_t \mid^2 + 2\,\Re e[d\rho_o{}^\star B_t{}^\star] + \mid \Delta_o \mid^2 R_s \\ g_2 =&\ \mid d \mid^2 + 2\,\Re e[d\rho_o{}^\star] + 2g_t\, \Re e[B_t] + 2\,\Re e[\Delta_o C_t{}^\star]\, R_s \\ g_3 =&\ -2\,(\Im m[B_t g_t + d\rho_o{}^\star] + \Im m[\Delta_o C_t{}^\star]\, R_s) \\ g_4 =&\ \mid C_t \mid^2 \\ c_1 =&\ g_t a^\star + \rho_o C_t{}^\star \end{aligned}$$

TABLE I
DUALITY RULES

I	R_n	ρ_n	Z_s	R_s	X_s	A_t	C_t	R_t	ρ_o
II	g_n	$\rho_n{}^\star$	Y_p	G_p	B_p	D_t	B_t	g_t	$\rho_o{}^\star$

$$
\begin{aligned}
c_2 &= g_t a^\star B_t + \rho_o C_t{}^\star B_t + \rho_o{}^\star a^\star d + R_t d C_t{}^\star \\
c_3 &= \rho_o B_t + R_t d \\
c_4 &= -d\Delta_o{}^\star \\
c_5 &= -\Delta_o a^\star \\
c_6 &= -B_t{}^\star d \\
c_7 &= -C_t a^\star \\
a &= 1 - A_t \\
d &= 1 - D_t \\
\Delta &= 1 + C_t Z_s + B_t Y_p + \Delta_o Y_p Z_s \\
\Delta_o &= 1 - a - d - (A_t D_t - B_t C_t) \\
\rho_o &= \rho_t \sqrt{R_t g_t}.
\end{aligned}
$$

III. THE DUALITY IN THE NOISE PARAMETERS

Equations (6)–(8) are ratios of polynomials where the common denominator is $\mid \Delta \mid^2$. Notice that the coefficients r_i of (6) depend on G_p, the real part of Y_p, but not on B_p, its imaginary part. Since R_n depends on B_p only through the denominator $\mid \Delta \mid^2$, it follows that a large value of susceptive feedback (at constant frequency) will decrease R_n. This dependence on B_p will make R_n close to zero for large values of $\mid Z_s \mid$ and different from zero for small values of $\mid Z_s \mid$ at constant Y_p.

Also notable is that the noise parameters transform into each other according to the rules of Table I. This set of duality rules is to be read as follows: if R_n is determined as in (6) but g_n has not yet been determined, then (7) can be worked out by substituting every symbol of (6) found in Table I, line I, with the corresponding one in line II. On this basis, if a particular behavior is found in R_n (g_n), a similar behavior will be expected in g_n (R_n).

IV. MINIMA IN R_n AND g_n

The noise parameters (6), (7), and (8) can be studied analytically. This aims to design $F_n \simeq F_{n_{\min}}$, an overall noise figure F_n as insensitive as possible to the mismatch $\mid Y_S - Y_{S\mathrm{opt}} \mid$ [12] or equivalently to $\mid \Gamma_S - \Gamma_{S\mathrm{opt}} \mid$. This goal can be achieved when R_n is as small as possible at the design frequency. Thus, the feedback element values which provide minima in R_n are sought. On the basis of the duality principle, equivalent results can be expected from g_n.

This analysis is easily carried out when one single reactive feedback element is considered. Thus, assume $Z_s = jX_s, Y_p = 0$; the dual task concerning g_n would require $Z_s = 0, Y_p = jB_p$.

In order to proceed, R_n is rewritten as

$$R_n = \frac{A\,X_s{}^2 + B\,X_s + C}{D\,X_s{}^2 + E\,X_s + 1}. \tag{9}$$

$A, \cdots, E$ are derived from (6). Since R_n cannot be negative, the following statements are satisfied.

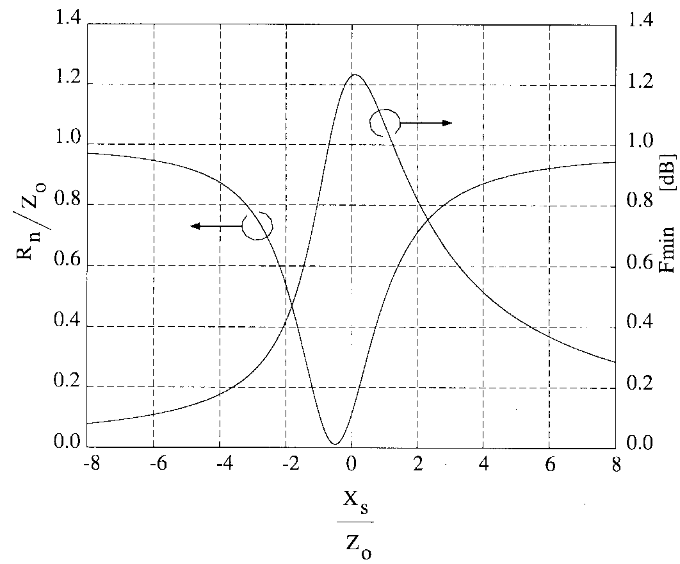

Fig. 2. Equivalent noise resistance R_n and minimum noise figure F_{min} versus feedback for Hewlett Packard ATF21186 MESFET at 8 GHz: $R_{n_{\max}} = 49.53\ \Omega$ at $C_s = 0.01$ pF (not shown); $R_{n_{\min}} = 0.57\ \Omega$ at $C_s = 0.80$ pF; $R_{n_{\mathrm{sat}}} = 49.42\ \Omega$; $Z_o = 50\ \Omega$.

1) The coefficient A is always positive:

$$A = \mid a\,\sqrt{g_t} + C_t\,\rho_t{}^\star\,\sqrt{R_t}\,\mid^2 + R_t \mid C_t \mid^2 \left(1 - \mid \rho_t \mid^2\right);$$

2) $B^2 - 4AC \leq 0$. Thus, a particular black box along with the proper feedback might provide $R_n = 0$.

The limit $R_{n_{\mathrm{sat}}} = \lim_{X_s \to \pm\infty} R_n$ is finite and positive because (9) is a ratio of second-degree polynomials.

By setting the first derivative to zero, it is found that the minima X_{s_o} satisfy

$$(AE - BD)\,X_{s_o}^2 + 2\,(A - DC)\,X_{s_o} + (B - EC) = 0.$$

Two solutions are expected: a minimum $R_{n_{\min}}$ for $X_{s_o} = X_{s_m}$ and a maximum $R_{n_{\max}}$ for $X_{s_o} = X_{s_M}$.

V. DISCUSSION OF THE RESULTS

Microwave-active devices such as MESFET's, JFET's, and HEMT's with a reactive-series feedback have been analyzed in order to work out the value of X_{s_m} and the corresponding $R_{n_{\min}}$. The simulation shows that where $R_{n_{\min}}$ occurs, the minimum noise figure $F_{n_{\min}}$ of the overall network is smaller than the minimum noise figure $F_{t_{\min}}$ of the black box. It is also noticeable that $R_{n_{\min}}$ may be achieved by a capacitive-series feedback (Fig. 2).

This analysis shows that if the signal matrix is comprised of real numbers, no minimum will occur. Thus, a microwave active device will exhibit a minimum, while a simple resistive attenuator will not. However, a minimum in the noise parameters will occur when feedback is applied to a passive *L*, *C*, *R* network.

$$R_n = \frac{r_1 \mid Z_s \mid^2 + r_2\,R_s + r_3\,X_s + r_4\,G_p + R_t}{\mid \Delta \mid^2} \tag{6}$$

$$g_n = \frac{g_1 \mid Y_p \mid^2 + g_2\,G_p + g_3\,B_p + g_4\,R_s + g_t}{\mid \Delta \mid^2} \tag{7}$$

$$\rho_n \sqrt{R_n g_n} = \frac{c_1 Z_s{}^\star + c_2 Z_s{}^\star Y_p + c_3 Y_p + c_4 Z_s{}^\star G_p + c_5 Y_p R_s + c_6 G_p + c_7 R_s + \rho_o}{\mid \Delta \mid^2} \tag{8}$$

TABLE II
CALCULATED RESULTS FOR THE DEVICE IN [13].

f	[GHz]	4	8
X_{s_m}	[Ω]	73.76	18.96
$R_{n_{min}}$	[Ω]	13.25	27.21
X_{s_M}	[Ω]	−116.51	−358.19
$R_{n_{max}}$	[Ω]	218.54	56.17
$R_{n_{sat}}$	[Ω]	145.89	55.96

To demonstrate experimental evidence for the validity of the analysis presented above, the results in [13] are considered, which show a minimum in R_n [13, Fig. 4]. If device parameters [13, page 324] are entered into (6), (7), and (8), the values of Table II are obtained in agreement with those results. The maximum in R_n is missing in [13, Fig. 4], since it occurs for a very large value of $(-X_s)$, where $R_n \simeq R_{n_{\max}} \simeq R_{n_{\text{sat}}}$.

VI. CONCLUSION

Closed-form expressions have been presented for the noise parameters with parallel and series feedback. It has been demonstrated that R_n always reaches a maximum and minimum, and the possibility of $R_n = 0$ has been pointed out. The same conclusions can be applied to g_n, since a duality principle exists. The theory shows that a minimum in the noise parameter R_n or g_n of either an active or passive black box may exist as long as its signal matrix is not purely real. A previous paper and its results have been used in order to demonstrate experimental evidence for the correctness of the formulas presented. This theory may help to design very low noise-feedback microwave amplifiers.

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Investigating Nonlinear Propagation in Dielectric Slab Waveguides

Jian-Guo Ma

***Abstract*—A numerical method is employed to analyze the TE-wave propagation in Kerr-like nonlinear dielectric waveguides in which a nonlinear film is sandwiched between two linear media. The dispersion curves dependent on the magnitude of the electric field are obtained. All the results can be used in future investigations of devices composed of nonlinear dielectric slab structures.**

***Index Terms*—Dispersion, Kerr-like, nonlinearity, waveguide.**

I. INTRODUCTION

It has been apparent for a long time that nonlinear propagation in optical and millimetric waveguides holds promise in the context of integrated signal processing [1]. In recent years, with the development of technology, guided waves in nonlinear dielectric slab waveguides received considerable attention owing to their potential applications to optical communications and optical computing.

For the nonlinear core waveguide, a general dispersion equation was developed in [2], using the modulus of a Jacobian elliptic function; however, spurious roots then appear in the dispersion equations [4]. The phase-plane approach was recently used in [1] to discuss the problem, which provides a physical interpretation of the results. This method can be applied to arbitrary nonlinearities. In all other cases, numerical methods such as in [3], [7], and [8], along with many others, have been employed.

In this paper, another numerical method is used to solve the nonlinear propagation in slab guides with a nonlinear core. The method transmits the values of the field from one boundary to another, therefore, it is called the transfer matrix method (TMM). In [9], the same idea was successfully used to numerically analyze the nonlinear planar waveguide with a linear core—a linear film is supported by a linear medium and covered by a nonlinear medium. In this paper, global coordinates are used to simplify the problem.

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